

# Package: ActuarialM (via r-universe)

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**Type** Package

**Title** Computation of Actuarial Measures Using Bell G Family

**Version** 0.1.0

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**Depends** R (>= 2.0)

**Imports** stats

**Description** It computes two frequently applied actuarial measures, the expected shortfall and the value at risk. Seven well-known classical distributions in connection to the Bell generalized family are used as follows: Bell-exponential distribution, Bell-extended exponential distribution, Bell-Weibull distribution, Bell-extended Weibull distribution, Bell-Lomax distribution, Bell-Burr-12 distribution, and Bell-Burr-X distribution. Related works include: a) Fayomi, A., Tahir, M. H., Algarni, A., Imran, M., & Jamal, F. (2022). ``A new useful exponential model with applications to quality control and actuarial data". Computational Intelligence and Neuroscience, 2022. <doi:10.1155/2022/2489998>. b) Alsadat, N., Imran, M., Tahir, M. H., Jamal, F., Ahmad, H., & Elgarhy, M. (2023). ``Compounded Bell-G class of statistical models with applications to COVID-19 and actuarial data". Open Physics, 21(1), 20220242. <doi:10.1515/phys-2022-0242>.

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ActuarialM-package	<i>Computation of Actuarial Measures Using Bell G Family</i>
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### Description

Evaluates the value at risk (VaR) and expected shortfall (ES) of seven well-known probability distributions in connection with the Bell G family of distributions.

### Details

Package: ActuarialM  
 Type: Package  
 Version: 0.1.0  
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### Maintainers

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### Author(s)

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### References

Fayomi, A., Tahir, M. H., Algarni, A., Imran, M., & Jamal, F. (2022). A new useful exponential model with applications to quality control and actuarial data. *Computational Intelligence and Neuroscience*, 2022. <doi:10.1155/2022/2489998>.

Alsadat, N., Imran, M., Tahir, M. H., Jamal, F., Ahmad, H., & Elgarhy, M. (2023). Compounded Bell-G class of statistical models with applications to COVID-19 and actuarial data. *Open Physics*, 21(1), 20220242. <doi:10.1155/2022/2489998>.

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 BellB12 distribution *Bell Burr-12 distribution*


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### Description

Computes the value at risk and expected shortfall based on the Bell Burr-12 (BellB12) distribution. The CDF of the Bell G family is as follows:

$$H(x) = \frac{1 - \exp[-e^\lambda (1 - e^{-\lambda K(x)})]}{1 - \exp(1 - e^\lambda)}; \quad \lambda > 0,$$

where  $K(x)$  represents the baseline Burr-12 CDF, it is given by

$$K(x) = 1 - \left[1 + \left(\frac{x}{a}\right)^b\right]^{-k}; \quad a, b, k > 0.$$

By setting  $K(x)$  in the above Equation, yields the CDF of the BellB12 distribution. The following expression can be used to calculate the VaR:

$$VaR_p(X) = a \left( \left[ \left( \frac{1}{\lambda} \left[ \ln \left( \left[ \ln \left( 1 - p \left[ 1 - \exp(1 - e^\lambda) \right] \right) \right] + e^\lambda \right) \right] \right]^{-1/k} - 1 \right)^{1/b},$$

where  $p \in (0, 1)$ . The ES can be computed from the following expression:

$$ES_p(X) = \frac{a}{p} \int_0^p \left( \left[ \left( \frac{1}{\lambda} \left[ \ln \left( \left[ \ln \left( 1 - z \left[ 1 - \exp(1 - e^\lambda) \right] \right) \right] + e^\lambda \right) \right] \right]^{-1/k} - 1 \right)^{1/b} dz.$$

### Usage

```
vBellB12(p, a, b, k, lambda, log.p = FALSE, lower.tail = TRUE)
eBellB12(p, a, b, k, lambda)
```

### Arguments

<code>p</code>	A vector of probabilities $p \in (0, 1)$ .
<code>lambda</code>	The strictly positive parameter of the Bell G family ( $\lambda > 0$ ).
<code>a</code>	The strictly positive scale parameter of the baseline Burr-12 distribution ( $a > 0$ ).
<code>b</code>	The strictly positive shape parameter of the baseline Burr-12 distribution ( $b > 0$ ).
<code>k</code>	The strictly positive shape parameter of the baseline Burr-12 distribution ( $k > 0$ ).
<code>lower.tail</code>	if FALSE then $1-H(x)$ are returned and quantiles are computed for $1-p$ .
<code>log.p</code>	if TRUE then $\log(H(x))$ are returned and quantiles are computed for $\exp(p)$ .

**Details**

The functions allow to compute the value at risk and the expected shortfall of the BellB12 distribution.

**Value**

vBellB12 gives the value at risk. eBellB12 gives the expected shortfall.

**Author(s)**

Muhammad Imran and M.H. Tahir.

R implementation and documentation: Muhammad Imran <imranshako0r84@yahoo.com> and M.H. Tahir <mht@iub.edu.pk>.

**References**

Fayomi, A., Tahir, M. H., Algarni, A., Imran, M., & Jamal, F. (2022). A new useful exponential model with applications to quality control and actuarial data. *Computational Intelligence and Neuroscience*, 2022.

Alsadat, N., Imran, M., Tahir, M. H., Jamal, F., Ahmad, H., & Elgarhy, M. (2023). Compounded Bell-G class of statistical models with applications to COVID-19 and actuarial data. *Open Physics*, 21(1), 20220242.

Zimmer, W. J., Keats, J. B., & Wang, F. K. (1998). The Burr XII distribution in reliability analysis. *Journal of quality technology*, 30(4), 386-394.

**See Also**

[eBellBX](#), [eBellL](#)

**Examples**

```
p=runif(10,min=0,max=1)
vBellB12(p,1,1,2,1.2)
eBellB12(p,1,1,2,1.2)
```

---

BellBX distribution    *Bell Burr-X distribution*

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**Description**

Computes the value at risk and expected shortfall based on the Bell Burr-X (BellBX) distribution. The CDF of the Bell G family is as follows:

$$H(x) = \frac{1 - \exp[-e^\lambda (1 - e^{-\lambda K(x)})]}{1 - \exp(1 - e^\lambda)}; \quad \lambda > 0,$$

where  $K(x)$  represents the baseline Burr-X CDF, it is given by

$$K(x) = [1 - \exp(-x^2)]^a; \quad a > 0.$$

By setting  $K(x)$  in the above Equation, yields the CDF of the BellBX distribution. The following expression can be used to calculate the VaR:

$$VaR_p(X) = \left( -\ln \left[ 1 - \left\{ 1 - \left( \frac{1}{\lambda} \left[ \ln \left( \left[ \ln \left( 1 - p \left[ 1 - \exp(1 - e^\lambda) \right] \right) \right] + e^\lambda \right) \right] \right\}^{1/a} \right] \right)^{0.5},$$

where  $p \in (0, 1)$ . The ES can be computed from the following expression:

$$ES_p(X) = \frac{1}{p} \int_0^p \left( -\ln \left[ 1 - \left\{ 1 - \left( \frac{1}{\lambda} \left[ \ln \left( \left[ \ln \left( 1 - z \left[ 1 - \exp(1 - e^\lambda) \right] \right) \right] + e^\lambda \right) \right] \right\}^{1/a} \right] \right)^{0.5} dz.$$

### Usage

```
vBellBX(p, a, lambda, log.p = FALSE, lower.tail = TRUE)
eBellBX(p, a, lambda)
```

### Arguments

<code>p</code>	A vector of probabilities $p \in (0, 1)$ .
<code>lambda</code>	The strictly positive parameter of the Bell G family ( $\lambda > 0$ ).
<code>a</code>	The strictly positive scale parameter of the baseline Burr-X distribution ( $a > 0$ ).
<code>lower.tail</code>	if FALSE then $1-H(x)$ are returned and quantiles are computed for $1-p$ .
<code>log.p</code>	if TRUE then $\log(H(x))$ are returned and quantiles are computed for $\exp(p)$ .

### Details

The functions allow to compute the value at risk and the expected shortfall of the BellBX distribution.

### Value

`vBellBX` gives the value at risk. `eBellBX` gives the expected shortfall.

### Author(s)

Muhammad Imran and M.H. Tahir.

R implementation and documentation: Muhammad Imran <imranshako0r84@yahoo.com> and M.H. Tahir <mht@iub.edu.pk>.

## References

- Fayomi, A., Tahir, M. H., Algarni, A., Imran, M., & Jamal, F. (2022). A new useful exponential model with applications to quality control and actuarial data. *Computational Intelligence and Neuroscience*, 2022.
- Alsadat, N., Imran, M., Tahir, M. H., Jamal, F., Ahmad, H., & Elgarhy, M. (2023). Compounded Bell-G class of statistical models with applications to COVID-19 and actuarial data. *Open Physics*, 21(1), 20220242.
- Kleiber, C., & Kotz, S. (2003). *Statistical size distributions in economics and actuarial sciences*. John Wiley & Sons.

## See Also

[eBellB12](#), [eBellL](#)

## Examples

```
p=runif(10,min=0,max=1)
vBellBX(p,1.2,2)
eBellBX(p,1.2,2)
```

---

Belle distribution      *Bell exponential distribution*

---

## Description

Computes the value at risk and expected shortfall based on the Bell exponential (Belle) distribution. The CDF of the Bell G family is as follows:

$$H(x) = \frac{1 - \exp[-e^\lambda (1 - e^{-\lambda K(x)})]}{1 - \exp(1 - e^\lambda)}; \quad \lambda > 0,$$

where  $K(x)$  represents the baseline exponential CDF, it is given by

$$K(x) = 1 - \exp(-\alpha x); \quad \alpha > 0.$$

By setting  $K(x)$  in the above Equation, yields the CDF of the Belle distribution. The following expression can be used to calculate the VaR:

$$VaR_p(X) = \frac{-1}{\alpha} \ln \left( \frac{1}{\lambda} \left\{ \ln \left[ \ln \left( 1 - p \left\{ 1 - \exp(1 - e^\lambda) \right\} \right) + e^\lambda \right] \right\} \right); \quad p \in (0, 1).$$

The ES can be computed from the following expression:

$$ES_p(X) = \frac{1}{p} \int_0^p \left[ \frac{-1}{\alpha} \ln \left( \frac{1}{\lambda} \left\{ \ln \left[ \ln \left( 1 - z \left\{ 1 - \exp(1 - e^\lambda) \right\} \right) + e^\lambda \right] \right\} \right) \right] dz.$$

**Usage**

```
vBelle(p, alpha, lambda, log.p = FALSE, lower.tail = TRUE)
eBelle(p, alpha, lambda)
```

**Arguments**

<code>p</code>	A vector of probabilities $p \in (0, 1)$ .
<code>lambda</code>	The strictly positive parameter of the Bell G family of distributions $\lambda > 0$ .
<code>alpha</code>	The strictly positive scale parameter of the baseline exponential distribution ( $\alpha > 0$ ).
<code>lower.tail</code>	if FALSE then $1-H(x)$ are returned and quantiles are computed for $1-p$ .
<code>log.p</code>	if TRUE then $\log(H(x))$ are returned and quantiles are computed for $\exp(p)$ .

**Details**

The functions allow to compute the value at risk and the expected shortfall of the Belle distribution.

**Value**

`vBelle` gives the values at risk. `eBelle` gives the expected shortfall.

**Author(s)**

Muhammad Imran and M.H. Tahir.

R implementation and documentation: Muhammad Imran <imranshako0r84@yahoo.com> and M.H. Tahir <mht@iub.edu.pk>.

**References**

Fayomi, A., Tahir, M. H., Algarni, A., Imran, M., & Jamal, F. (2022). A new useful exponential model with applications to quality control and actuarial data. *Computational Intelligence and Neuroscience*, 2022.

Alsadat, N., Imran, M., Tahir, M. H., Jamal, F., Ahmad, H., & Elgarhy, M. (2023). Compounded Bell-G class of statistical models with applications to COVID-19 and actuarial data. *Open Physics*, 21(1), 20220242.

Nadarajah, S. (2011). The exponentiated exponential distribution: a survey. *AStA Advances in Statistical Analysis*, 95, 219-251.

**See Also**

[eBellW](#), [eBellEE](#)

**Examples**

```
p=runif(10,min=0,max=1)
vBelle(p,1,1.2)
eBelle(p,1,1.2)
```

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BellEE distribution    *Bell exponentiated exponential distribution*

---

### Description

Computes the value at risk and expected shortfall based on the Bell exponentiated exponential (BellEE) distribution. The CDF of the Bell G family is as follows:

$$H(x) = \frac{1 - \exp[-e^\lambda (1 - e^{-\lambda K(x)})]}{1 - \exp(1 - e^\lambda)}; \quad \lambda > 0,$$

where  $K(x)$  represents the baseline exponentiated exponential CDF, it is given by

$$K(x) = [1 - \exp(-\alpha x)]^\beta; \quad \alpha, \beta > 0.$$

By setting  $K(x)$  in the above Equation, yields the CDF of the BellEE distribution. The following expression can be used to calculate the VaR:

$$VaR_p(X) = \frac{-1}{\alpha} \ln \left[ 1 - \left( 1 - \left( \frac{1}{\lambda} \left[ \ln \left( \left[ \ln \left( 1 - p \left[ 1 - \exp(1 - e^\lambda) \right] \right) \right] + e^\lambda \right) \right) \right)^{1/\beta} \right],$$

where  $p \in (0, 1)$ . The ES can be computed from the following expression:

$$ES_p(X) = \frac{1}{p} \int_0^p \left[ \frac{-1}{\alpha} \ln \left[ 1 - \left( 1 - \left( \frac{1}{\lambda} \left[ \ln \left( \left[ \ln \left( 1 - z \left[ 1 - \exp(1 - e^\lambda) \right] \right) \right] + e^\lambda \right) \right) \right)^{1/\beta} \right] \right] dz.$$

### Usage

```
vBellEE(p, alpha, beta, lambda, log.p = FALSE, lower.tail = TRUE)
eBellEE(p, alpha, beta, lambda)
```

### Arguments

<code>p</code>	A vector of probabilities $p \in (0, 1)$ .
<code>lambda</code>	The strictly positive parameter of the Bell G family of distributions $\lambda > 0$ .
<code>alpha</code>	The strictly positive scale parameter of the baseline exponentiated exponential distribution ( $\alpha > 0$ ).
<code>beta</code>	The strictly positive shape parameter of the baseline exponentiated exponential distribution ( $\beta > 0$ ).
<code>lower.tail</code>	if FALSE then $1-H(x)$ are returned and quantiles are computed for $1-p$ .
<code>log.p</code>	if TRUE then $\log(H(x))$ are returned and quantiles are computed for $\exp(p)$ .

### Details

The functions allow to compute the value at risk and the expected shortfall of the BellEE distribution.



**Value**

vBellEE gives the value at risk. eBellEE gives the expected shortfall.

**Author(s)**

Muhammad Imran and M.H. Tahir.

R implementation and documentation: Muhammad Imran <imranshako0r84@yahoo.com> and M.H. Tahir <mht@iub.edu.pk>.

**References**

Fayomi, A., Tahir, M. H., Algarni, A., Imran, M., & Jamal, F. (2022). A new useful exponential model with applications to quality control and actuarial data. Computational Intelligence and Neuroscience, 2022.

Alsadat, N., Imran, M., Tahir, M. H., Jamal, F., Ahmad, H., & Elgarhy, M. (2023). Compounded Bell-G class of statistical models with applications to COVID-19 and actuarial data. Open Physics, 21(1), 20220242.

Nadarajah, S. (2011). The exponentiated exponential distribution: a survey. AStA Advances in Statistical Analysis, 95, 219-251.

**See Also**

[eBelleW](#), [eBelleE](#)

**Examples**

```
p=runif(10,min=0,max=1)
vBellEE(p,1,1.2,2)
eBellEE(p,1,1.2,2)
```

BelleW distribution     *Bell exponentiated Weibull distribution*

**Description**

Computes the value at risk and expected shortfall based on the Bell exponentiated Weibull (BelleW) distribution. The CDF of the Bell G family is as follows:

$$H(x) = \frac{1 - \exp[-e^\lambda (1 - e^{-\lambda K(x)})]}{1 - \exp(1 - e^\lambda)}; \quad \lambda > 0,$$

where K(x) represents the baseline exponentiated Weibull CDF, it is given by

$$K(x) = [1 - \exp(-\alpha x^\beta)]^\theta; \quad \alpha, \beta, \theta > 0.$$

By setting  $K(x)$  in the above Equation, yields the CDF of the BelleW distribution. The following expression can be used to calculate the VaR:

$$VaR_p(X) = \left[ \frac{-1}{\alpha} \ln \left( 1 - \left[ 1 - \left( \frac{1}{\lambda} \left[ \ln \left( \left[ \ln \left( 1 - p \left[ 1 - \exp(1 - e^\lambda) \right] \right) \right] + e^\lambda \right) \right] \right)^{1/\theta} \right] \right)^{1/\beta},$$

where  $p \in (0, 1)$ . The ES can be computed from the following expression:

$$ES_p(X) = \frac{1}{p} \int_0^p \left[ \frac{-1}{\alpha} \ln \left( 1 - \left[ 1 - \left( \frac{1}{\lambda} \left[ \ln \left( \left[ \ln \left( 1 - z \left[ 1 - \exp(1 - e^\lambda) \right] \right) \right] + e^\lambda \right) \right] \right)^{1/\theta} \right] \right)^{1/\beta} dz.$$

### Usage

```
vBelleW(p, alpha, beta, theta, lambda, log.p = FALSE, lower.tail = TRUE)
eBelleW(p, alpha, beta, theta, lambda)
```

### Arguments

<code>p</code>	A vector of probabilities $p \in (0, 1)$ .
<code>lambda</code>	The strictly positive parameter of the Bell G family of distributions $\lambda > 0$ .
<code>alpha</code>	The strictly positive scale parameter of the baseline exponentiated Weibull distribution ( $\alpha > 0$ ).
<code>beta</code>	The strictly positive shape parameter of the baseline exponentiated Weibull distribution ( $\beta > 0$ ).
<code>theta</code>	The strictly positive shape parameter of the baseline exponentiated Weibull distribution ( $\theta > 0$ ).
<code>lower.tail</code>	if FALSE then $1-H(x)$ are returned and quantiles are computed for $1-p$ .
<code>log.p</code>	if TRUE then $\log(H(x))$ are returned and quantiles are computed for $\exp(p)$ .

### Details

The functions allow to compute the value at risk and the expected shortfall of the BelleW distribution.

### Value

`vBelleW` gives the value at risk. `eBelleW` gives the expected shortfall.

### Author(s)

Muhammad Imran and M.H. Tahir.

R implementation and documentation: Muhammad Imran <imranshakoor84@yahoo.com> and M.H. Tahir <mht@iub.edu.pk>.

## References

- Fayomi, A., Tahir, M. H., Algarni, A., Imran, M., & Jamal, F. (2022). A new useful exponential model with applications to quality control and actuarial data. *Computational Intelligence and Neuroscience*, 2022.
- Alsadat, N., Imran, M., Tahir, M. H., Jamal, F., Ahmad, H., & Elgarhy, M. (2023). Compounded Bell-G class of statistical models with applications to COVID-19 and actuarial data. *Open Physics*, 21(1), 20220242.
- Nadarajah, S., Cordeiro, G. M., & Ortega, E. M. (2013). The exponentiated Weibull distribution: a survey. *Statistical Papers*, 54, 839-877.

## See Also

[eBellW](#), [eBellEE](#)

## Examples

```
p=runif(10,min=0,max=1)
vBellEW(p,1,1,2,1)
eBellEW(p,1,1,2,1)
```

---

BellL distribution      *Bell Lomax distribution*

---

## Description

Computes the value at risk and expected shortfall based on the Bell Lomax (BellL) distribution. The CDF of the Bell G family is as follows:

$$H(x) = \frac{1 - \exp[-e^\lambda (1 - e^{-\lambda K(x)})]}{1 - \exp(1 - e^\lambda)}; \quad \lambda > 0,$$

where  $K(x)$  represents the baseline Lomax CDF, it is given by

$$K(x) = 1 - \left[1 + \left(\frac{x}{b}\right)\right]^{-q}; \quad b, q > 0.$$

By setting  $K(x)$  in the above Equation, yields the CDF of the BellL distribution. The following expression can be used to calculate the VaR:

$$VaR_p(X) = b \left[ \left( \frac{1}{\lambda} \left[ \ln \left( \left[ \ln \left( 1 - p \left[ 1 - \exp(1 - e^\lambda) \right] \right) \right] + e^\lambda \right) \right]^{-1/q} - 1 \right) \right],$$

where  $p \in (0, 1)$ . The ES can be computed from the following expression:

$$ES_p(X) = \frac{b}{p} \int_0^p \left[ \left( \frac{1}{\lambda} \left[ \ln \left( \left[ \ln \left( 1 - z \left[ 1 - \exp(1 - e^\lambda) \right] \right) \right] + e^\lambda \right) \right]^{-1/q} - 1 \right) \right] dz.$$

**Usage**

```
vBellL(p, b, q, lambda, log.p = FALSE, lower.tail = TRUE)
eBellL(p, b, q, lambda)
```

**Arguments**

p	A vector of probabilities $p \in (0, 1)$ .
lambda	The strictly positive parameter of the Bell G family ( $\lambda > 0$ ).
b	The strictly positive scale parameter of the baseline Lomax distribution ( $b > 0$ ).
q	The strictly positive shape parameter of the baseline Lomax distribution ( $q > 0$ ).
lower.tail	if FALSE then $1-H(x)$ are returned and quantiles are computed for $1-p$ .
log.p	if TRUE then $\log(H(x))$ are returned and quantiles are computed for $\exp(p)$ .

**Details**

The functions allow to compute the value at risk and the expected shortfall of the BellL distribution.

**Value**

vBellL gives the values at risk. eBellL gives the expected shortfall.

**Author(s)**

Muhammad Imran and M.H. Tahir.

R implementation and documentation: Muhammad Imran <imranshako0r84@yahoo.com> and M.H. Tahir <mht@iub.edu.pk>.

**References**

Fayomi, A., Tahir, M. H., Algarni, A., Imran, M., & Jamal, F. (2022). A new useful exponential model with applications to quality control and actuarial data. *Computational Intelligence and Neuroscience*, 2022.

Alsadat, N., Imran, M., Tahir, M. H., Jamal, F., Ahmad, H., & Elgarhy, M. (2023). Compounded Bell-G class of statistical models with applications to COVID-19 and actuarial data. *Open Physics*, 21(1), 20220242.

Kleiber, C., & Kotz, S. (2003). *Statistical size distributions in economics and actuarial sciences*. John Wiley & Sons.

**See Also**

[eBellBX](#), [eBellB12](#)

**Examples**

```
p=runif(10,min=0,max=1)
vBellL(p,1,1,2)
eBellL(p,1,1,2)
```

---

BellW distribution      *Bell Weibull distribution*

---

### Description

Computes the value at risk and expected shortfall based on the Bell Weibull (BellW) distribution. The CDF of the Bell G family is as follows:

$$H(x) = \frac{1 - \exp[-e^\lambda (1 - e^{-\lambda K(x)})]}{1 - \exp(1 - e^\lambda)}; \quad \lambda > 0,$$

where  $K(x)$  represents the baseline Weibull CDF, it is given by

$$K(x) = 1 - \exp(-\alpha x^\beta); \quad \alpha, \beta > 0.$$

By setting  $K(x)$  in the above Equation, yields the CDF of the BellW distribution. The following expression can be used to calculate the VaR:

$$VaR_p(X) = \left[ \frac{-1}{\alpha} \ln \left( \frac{1}{\lambda} \{ \ln [ \ln (1 - p \{ 1 - \exp(1 - e^\lambda) \}) + e^\lambda ] \} \right) \right]^{1/\beta}; \quad p \in (0, 1).$$

The ES can be computed from the following expression:

$$ES_p(X) = \frac{1}{p} \int_0^p \left[ \frac{-1}{\alpha} \ln \left( \frac{1}{\lambda} \{ \ln [ \ln (1 - z \{ 1 - \exp(1 - e^\lambda) \}) + e^\lambda ] \} \right) \right]^{1/\beta} dz.$$

### Usage

```
vBellW(p, alpha, beta, lambda, log.p = FALSE, lower.tail = TRUE)
eBellW(p, alpha, beta, lambda)
```

### Arguments

<code>p</code>	A vector of probabilities $p \in (0, 1)$ .
<code>lambda</code>	The strictly positive parameter of the Bell G family of distributions $\lambda > 0$ .
<code>alpha</code>	The strictly positive scale parameter of the baseline Weibull distribution ( $\alpha > 0$ ).
<code>beta</code>	The strictly positive shape parameter of the baseline Weibull distribution ( $\beta > 0$ ).
<code>lower.tail</code>	if FALSE then $1-H(x)$ are returned and quantiles are computed for $1-p$ .
<code>log.p</code>	if TRUE then $\log(H(x))$ are returned and quantiles are computed for $\exp(p)$ .

### Details

The functions allow to compute the value at risk and the expected shortfall of the BellW distribution.

**Value**

vBellW gives the values at risk. eBellW gives the expected shortfall.

**Author(s)**

Muhammad Imran and M.H. Tahir.

R implementation and documentation: Muhammad Imran <imranshakoor84@yahoo.com> and M.H. Tahir <mht@iub.edu.pk>.

**References**

Fayomi, A., Tahir, M. H., Algarni, A., Imran, M., & Jamal, F. (2022). A new useful exponential model with applications to quality control and actuarial data. *Computational Intelligence and Neuroscience*, 2022.

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**See Also**

[eBelleW](#), [eBelleE](#)

**Examples**

```
p=runif(10,min=0,max=1)
vBellW(p,1,2,1)
eBellW(p,1,2,1)
```

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